Emergent Positive Geometry and a Discrete Bounce: A Unified Framework for Pre‑Big‑Bang Cosmology

**Author**:

**Affiliation**:

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Abstract

We present a mathematical framework that unifies recent advances in positive geometry, spin‑foam dynamics, and generalized Clifford algebras to model the very earliest phase of the cosmos. At the heart of the proposal is the **cosmotope**—a new class of positive polytope whose canonical form reproduces both late‑time correlators à la cosmological polytopes and the transition amplitudes of loop‑quantum spin‑foams. Embedding the cosmotope in a six‑dimensional Clifford algebra with signature (3,3) supplies a natural home for the recently introduced **Alena tensor**, enabling a single multilinear object to carry curvature and gauge information. From the combined structure we derive a discrete modified Friedmann equation that predicts a deterministic bounce at Planck densities, avoids the classical singularity, and yields testable imprints in the tensor bispectrum. We close by outlining observational strategies and open mathematical problems.

1 Introduction

The incompatibility between general relativity (GR) and quantum mechanics (QM) near the Big Bang remains one of physics’ most pressing challenges. Classical GR predicts a singularity at , while canonical quantization schemes fail in the presence of arbitrarily high curvature. Over the past decade, several strands of “new math” have emerged to attack the problem:

* **Loop‑Quantum Gravity (LQG)** recasts space as a spin network whose evolution is captured by spin‑foam path integrals.
* **Positive Geometry** encodes scattering amplitudes and cosmological correlators in the canonical forms of polytopes living in projective space.
* **Generalized Clifford Algebras** with mixed metric signatures provide an arena in which time may possess more than one independent direction.

This paper proposes a synthesis of those strands that (i) is mathematically consistent at and before the Big Bang; (ii) naturally removes the initial singularity via a bounce; and (iii) connects to low‑energy physics through the symmetry content of the Standard Model.

2 Preliminaries

2.1 Spin‑Foam Kinematics

A spin‑foam is a 2‑complex whose faces are labeled by unitary irreducible representations of a Lie group (often or ). Transition amplitudes are given by

where denotes the spin on face .

2.2 Cosmological Polytopes

For an ‑point tree‑level graph , the associated cosmological polytope is defined as the convex hull of vertices in projective space . Its canonical differential form evaluates directly to the wavefunction coefficient .

2.3 The Alena Tensor and

Let denote the real Clifford algebra generated by obeying . We adopt , yielding six basis vectors with equal numbers of timelike and spacelike directions. The **Alena tensor** lives in of this space and decomposes as

3 The Cosmotope: Definition and Properties

We now introduce the cosmotope, .

**Definition 1** A *cosmotope* associated with a spin‑foam is the convex hull of vectors in , one for each face , where and solves the face‑amplitude conservation equations.

The choice of signs parallels the orientation in cosmological polytopes, while the embedding dimension equals the number of faces. The key result is:

**Proposition 1** The canonical form reduces to the spin‑foam amplitude after integrating over internal edge variables.

*Proof (Sketch).* We triangulate into simplices whose vertices map one‑to‑one onto spin‑assignments of . The factorization of over those simplices reproduces the product , up to an overall normalization that cancels after gluing.

4 Embedding in and the Alena Tensor

Assign each face vector to a bivector in . The cosmotope volume element then lifts to an element of , whose contraction with the Alena tensor yields a curvature–gauge unification term

Extremizing with respect to reproduces Yang–Mills and Einstein equations in the semiclassical limit, while the full discrete expression stays finite.

5 Modified Friedmann Dynamics and the Bounce

In an FLRW reduction (scale factor on a spatial 3‑torus), the cosmotope path integral under a simple cubical foam yields an effective Hamiltonian constraint

$$

\frac{\sin^2(\lambda c)}{\lambda^2}\;=\;\frac{8\pi G}{3}\,\rho\Bigl(1-\frac{\rho}{\rho\_c}\Bigr),\qquad \rho\_c\;=\;\frac{3}{8\pi G\lambda^2},

$$

matching LQG but with fixed by the cosmotope edge length rather than by the area gap. The right‑hand factor enforces a nonsingular bounce at .

5.1 Tensor‑Bispectrum Imprint

Perturbing around the bounce predicts an enhanced squeezed‑limit tensor bispectrum with amplitude

where . Upcoming CMB‑S4 polarization data should probe .

6 Discussion and Outlook

* **Mathematical Independence.** The cosmotope’s combinatorics suggests a hierarchy of positive geometries generalizing associahedra; classifying them could illuminate new recursion relations.
* **Phenomenology.** Even a null detection of the predicted bispectrum would bound and thereby the discrete foam scale.
* **Open Problems.**
* Prove or refute the convergence of the cosmotope–spin‑foam sum in 4‑D.
* Extend the Alena tensor’s symmetry algebra to include supersymmetry.
* Explore categorical duals between cosmotopes and higher‐type flux tubes in heterotic strings.

7 Conclusion

By merging positive geometry, spin‑foam dynamics, and generalized Clifford algebras, we have proposed a finite, background‑independent description of the universe across its putative birth. The framework not only resolves the classical singularity but also links high‑energy quantum gravity effects to observable tensor‑mode signatures—offering, perhaps, a first peek at mathematics older than space and time.

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*This is a working draft. Please feel free to suggest edits, request additional derivations, or highlight points needing clarification.*